# A COMPARISON OF MACHINE LEARNING APPROACHES AND CLASSICAL NUMERICAL METHODS FOR THE RESOLUTION OF ELECTROMAGNETIC PROBLEMS

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### Abstract

Machine Learning (ML) is getting more and more present as an alternative computing approach for the numerical analysis of electromagnetic fields. On the other hand, specialised methods like Finite Elements (FEM) [1] or Boundary Elements (BEM) [2] proved to be quite effective in terms of accuracy and computational burden, yet requiring ad-hoc modelling for each different problem. In this contribution we propose a comparison among different ML approaches with some classical approaches in terms of accuracy, time to obtain the result, and generalization capabilities.

## 1 Introduction

The use of ML techniques in computational electromagnetism allows a very prompt evaluation of ElectroMagnetic (EM) fields when running an optimization process for some device or solving an inverse problem [3]. In both cases, repeated evaluations of similar configurations are required, and promptness is a major issue. The performance in terms of promptness is achieved at the expenses of a time-consuming training phase, when the ML model is adjusted to fit the required input-output data. In this phase also the time required to gather the training examples should be included, although Physically-Informed Neural Networks (PINN) [4] can significantly reduce this effort, as they train by minimizing the physical equations residual rather than the discrepancy with available data. A second issue related to use of ML for EM fields representation is about the accuracy, which is not generally high, especially when configurations not included in the training data set are referenced. Finally, the generalization capability of the model is a further issue to consider in the design of the approach. As a matter of fact, a trade-off between model complexity and its generalization capabilities must be considered, since including design parameters like topology or excitation among the inputs of the ML model required more complex models and larger training sets. Classical approaches to EM fields numerical computation have been studied since a long ago, examples being FEM, BEM, or Meshless approaches. The cited methods all fall under the more general class of "Moment-Method" based approaches [5]. They have been highly tailored for each physical problem, and the capabilities of each have been thoroughly assessed in terms of computational effort, accuracy and generalization capabilities.

We propose here a comparison between some ML approaches and FEM method. The different resolution methods will be compared on a few benchmark cases, highlighting different aspects of the typical EM devices study problems. In this digest we first present with some detail the different ML approaches and FEM method considered in the comparison, then we present the benchmark problems and the Key Performance Indicators (KPI) used to compare the methods, and finally, in this digest, some preliminary results.

## 2 Comparison of ML and Classical approaches

The aim of this work is to compare ML approaches and classical ones on the basis of the following three KPIs:

- $T_{EM}$: time to gather an estimate of the EM field – differentiating between training and computing times,
- $\Delta_A$: accuracy in the representation of the response – assessed with respect to a reference method,
- $\Delta_G$: generalization capabilities – assessed as the $\Delta_A$ on previously unforeseen cases.

The KPI depend on the complexity of the models, both for ML and classical ones, and their behaviours with respect to model complexity will also be investigated. In this digest, the reference method used to compute $\Delta_A$ is the FEM, while $T_{EM}$ and $\Delta_G$ are not reported here. Such a comparison is a quite ambitious, so we will limit ourselves to compare just a subset of the available methodologies. Specifically, we will consider the following ML approaches: Deep Neural Networks (DNNs) with equation residual minimization (so-called PINN), a Hybrid Boundary Element – Physics Informed Neural Network (BEM-PINN) method, recently proposed by the authors [6]. For classical approaches, we will consider the FEM method and Meshless method [7]. Different approaches are characterised by different sets of parameters: in classical methods they are the number of basis elements (corresponding to the discretization level), while in PINN they are both the number of layers/neurons and the number of grid points. The selection of the best PINN architecture can be considered equivalent to the selection of the proper mesh; in addition, training of the PINN is a process that is time consuming, and not comparable to any equivalent in classical methods. For this reason the first step is to compare the accuracy and the CPU time for a single run, ignoring the training time. We will use hyperbolic tangent (tanh) activation functions because these provide smooth, differentiable outputs, possess the symmetry with respect to zero, and finally allow achieving faster convergence. For the FEM, we will use second order functions on a simplicial mesh, while for the Meshless method Gaussian Basis functions will be used.

## 3 The benchmark problems

In order to test the different approaches on a "basic" problem, the benchmark will be the computation of the electric potential inside a L-shaped domain. Dirichlet boundary conditions are imposed on $\{(x = 0; 0 < y < 1) \cup (0 < x < 1; y = 0)\}$ where $\varphi = 1$, and on $\{(x = 0.5; 0.5 < y < 1) \cup (0.5 < x < 1; y = 0.5)\}$ where $\varphi = 0$. Neumann BC are imposed on the remaining part of the boundary $\{(0 < x < 0.5; y = 1) \cup (x = 1; 0 < y < 0.5)\}$. No charge is present in the domain in this case. Fig. 1 shows a representation of the domain, together with the reference solution, obtained using a FEM approach, and a mesh with 7500 degrees of freedom. In order to compute the KPI for the PINN method, a regular grid has been created by considering 100 points on the long side $\{0 < x < 0.5, 0 < y < 1\}$ and 50 on the short side of the L shaped domain $\{0.5 < x < 1, 0 < y < 0.5\}$. As for the BEM – PINN method, the same grid has been maintained in the subdomain where the PINN is used (left rectangle), while a 50 elements segmentation on the remaining parts of the boundary has been considered.

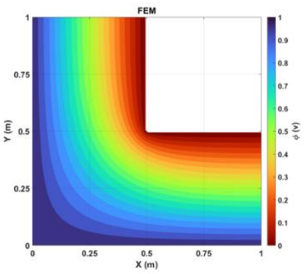


*Figure 1*. 2D potential map in an L-shaped domain, calculated using FEM.

## 4 Preliminary Results

To compare the DA for the PINN and BEM-PINN approaches, the 2D potential over the L-shaped domain obtained using BEM-PINN and PINN is subtracted from the FEM result. The two maps of DA are shown in Figure 2. As shown in the figure, the maximum deviation of the PINN approach from FEM occurs at the corner, where the accuracy of PINN decreases and where, in general all numerical methods fail to produce accurate results. However, the BEM-PINN method demonstrates higher accuracy, particularly at the corners. The deviation between BEM-PINN and FEM is lower across the entire domain, indicating the superior accuracy of this approach compared to PINN.

## 5 Conclusions

This study aims to highlighting the strengths and limitations of both ML and classical numerical methods for solving electromagnetic problems. While classical approaches like FEM offer high accuracy, they face significant computational demands as the mesh grid density increases. On the other hand, ML approaches, particularly PINN and BEM-PINN, provide mesh-free solutions with faster computational times once trained. Just a first preliminary assessment is shown in this digest, allowing to state that BEM-PINN outperforms PINN in terms of accuracy, especially in regions with complex geometries such as corners.

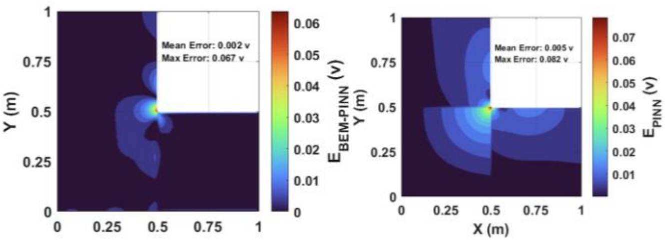


*Figure 2*. $\Delta_A$ for the BEM-PINN approach (left plot), and the PINN approach (right plot).

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